PHYSICS TOPICAL:

Light and Geometrical Optics
Test 1

Time: 23 Minutes*
Number of Questions: 18

* The timing restrictions for the science topical tests are optional. If you are using this test for the sole purpose of content reinforcement, you may want to disregard the time limit.
**DIRECTIONS:** Most of the questions in the following test are organized into groups, with a descriptive passage preceding each group of questions. Study the passage, then select the single best answer to each question in the group. Some of the questions are not based on a descriptive passage; you must also select the best answer to these questions. If you are unsure of the best answer, eliminate the choices that you know are incorrect, then select an answer from the choices that remain. Indicate your selection by blackening the corresponding circle on your answer sheet. A periodic table is provided below for your use with the questions.

**PERIODIC TABLE OF THE ELEMENTS**

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* Periodic Table of the Elements (Continued on Next Page)
Passage I (Questions 1–6)

Figure 1 shows a simplified model of the eye that is based on the assumption that all of the refraction of entering light occurs at the cornea. The cornea is a converging lens located at the outer surface of the eye with fixed focal length approximately equal to 2 cm. Parallel light rays coming from a very distant object are refracted by the cornea to produce a focused image on the retina. The retina then transmits electrical impulses along the optic nerve to the brain.

Two common defects of vision are myopia and hyperopia. Myopia, sometimes referred to as nearsightedness, occurs when the cornea focuses the image of a distant object in front of the retina. Hyperopia, sometimes referred to as farsightedness, occurs when the cornea focuses the image of a nearby object behind the retina. Both of these problems can be corrected by introducing another lens in front of the eye so that the two-lens system produces a focused image on the retina. If an object is so far away from the lens system that its distance may be taken as infinite, then the following relationship holds: $\frac{1}{f_c} + \frac{1}{f_l - x} = \frac{1}{i}$, where $f_c$ is the focal length of the cornea, $f_l$ is the focal length of the correcting lens, $x$ is the distance from the correcting lens to the cornea, and $i$ is the image distance measured from the cornea. (Note: The index of refraction is 1.0 for air and 1.5 for glass.)

1. How far away should the retina be from the cornea for normal vision?
   - A. 0.5 cm
   - B. 1.0 cm
   - C. 2.0 cm
   - D. 4.0 cm

2. For a distant object, the image produced by the cornea is:
   - A. real and inverted.
   - B. real and upright.
   - C. virtual and inverted.
   - D. virtual and upright.

3. What kind of lens would be suitable to correct myopia and hyperopia respectively? (Note: Assume that the correcting lens is at the focal point of the cornea so that $x = f_c$.)
   - A. Converging, converging
   - B. Converging, diverging
   - C. Diverging, diverging
   - D. Diverging, converging

4. The focal length of a woman’s cornea is 1.8 cm, and she wears a correcting lens with a focal length of \(-16.5\) cm at a distance $x = 1.5$ cm from her cornea. What is the image distance $i$ measured from the cornea for a distant object?
   - A. 1.0 cm
   - B. 1.5 cm
   - C. 2.0 cm
   - D. 2.5 cm

5. In the case of contact lenses, the cornea and the correcting lens are actually touching and act together as a single lens. If the focal length of both the cornea and the contact lens are doubled, then the image distance $i$ for a distant object would:
   - A. be $1/4$ the old value.
   - B. be $1/2$ the old value.
   - C. be the same as the old value.
   - D. be twice the old value.

6. Light bends towards the normal as it travels from air into a glass lens. This can be best explained by the fact that:
   - A. light travels slower in glass than in air.
   - B. light travels faster in glass than in air.
   - C. the speed of light is independent of the medium in which it travels.
   - D. some of the light is reflected at the surface of the glass lens.

GO ON TO THE NEXT PAGE.
8. A circularly polarized beam of light propagates through a vacuum with wavelength equal to 600 nm. What is the frequency of this wave?
A. $5 \times 10^{12}$ Hz  
B. $2 \times 10^{13}$ Hz  
C. $5 \times 10^{14}$ Hz  
D. $2 \times 10^{15}$ Hz

9. A plane polarized electromagnetic wave propagates with $E_{\text{rms}} = 30$ V/m. What is the power transmitted to a circular disk of radius $r = 2$ m, if all of the light is absorbed by the disk and $\mathbf{S}$ is perpendicular to the disk?
A. 10 J/s  
B. 30 J/s  
C. 60 J/s  
D. 90 J/s

10. $\lambda$ and $f$ are the respective wavelength and frequency of an electromagnetic wave traveling in a vacuum. Which of the following statements are true of the wave traveling in a medium having index of refraction $n$?
I. Its speed equals $c/n$.  
II. Its wavelength equals $\lambda/n$.  
III. Its frequency equals $f/n$.  
A. I only  
B. I and II only  
C. II and III only  
D. I, II, and III

11. A monochromatic electromagnetic wave propagates so that $\mathbf{S}$ points out of the page, $\mathbf{E}$ oscillates in the vertical direction, and $\mathbf{B}$ oscillates in the horizontal direction. If the light passes through a polarizing filter with horizontal polarizing direction, then it will:
A. have zero intensity.  
B. be polarized in the horizontal direction with half the intensity.  
C. be polarized in the horizontal direction with twice the intensity.  
D. be unpolarized with the same intensity.

12. When an electromagnetic wave is totally reflected by a surface, its change in momentum is double that when it is totally absorbed. The radiation pressure for total reflection, $p_r$, is therefore given by:
A. $p_r = I/2c$  
B. $p_r = I/c$  
C. $p_r = 2I/c$  
D. $p_r = 4I/c$
13. Which of the following can produce a real and inverted image?

I. A convex mirror
II. A concave mirror
III. A concave lens

A. I only
B. II only
C. I and II only
D. II and III only

14. A convex mirror has a focal point \( f \) as shown in the figure below. If a real object is at \( o \), then the correct image is given by:

![Diagram of a convex mirror with object at o and image at A, C, or D]

A. A
B. B
C. C
D. D

15. Light of wavelength 600 nm passes from air into a medium of higher density. The ratio of the index of refraction of the medium to that of air is known. What additional information is needed to determine the angle of reflection of the incident light at the boundary of the medium?

A. The wavelength of the light in the medium
B. The index of refraction of the medium
C. The angle of refraction of the incident light at the boundary of the medium
D. The density of the medium
16. Two polarizing lenses are aligned so that the intensity of transmitted light is at a maximum. If the first lens is rotated 45° in a clockwise direction, through what angle must the second lens be rotated in the counterclockwise direction so that the intensity of transmitted light is again at a maximum?

A. 30°
B. 45°
C. 90°
D. 135°

17. Stars as viewed through a refracting telescope often appear to be surrounded by blurry, rainbow-colored fringes. This can be explained by the fact that:

A. different colors of light travel at different speeds in a vacuum.
B. lenses bend different colors of light through different angles.
C. the stars are very far away.
D. the Earth’s atmosphere changes the apparent color of a star.

18. Light travels from medium 1 into medium 2, where the index of refraction, \( n_1 \), of medium 1 is greater than the index of refraction, \( n_2 \), of medium 2. In order for the light to be totally internally reflected at the boundary:

A. the angle of incidence must equal the angle of reflection.
B. the angle of refraction must equal the angle of reflection.
C. the angle of refraction must be greater than \( \sin^{-1}(n_1/n_2) \)
D. the angle of incidence must be greater than \( \sin^{-1}(n_2/n_1) \).
**ANSWER KEY:**

1. C  
2. A  
3. D  
4. C  
5. D  
6. A  
7. D  
8. C  
9. B  
10. B  
11. A  
12. C  
13. B  
14. A  
15. C  
16. D  
17. B  
18. D
EXPLANATIONS

Passage 1 (Questions 1—6)

1. C
   The passage states in the first paragraph that the focal length of the cornea is about 2 cm. We are also told that in a normal eye parallel light rays are refracted by the cornea to produce a focused image on the retina. Since the cornea is a converging lens, the parallel light rays from a distant object will be focused at its focal length. We can see this from the thin lens formula $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$, where o is the object distance, i is the image distance, and f the focal length of the lens. For a distant object we can set o equal to infinity, and $\frac{1}{o} = 0$. Therefore, the image distance equals the focal length. If the image is to be focused on the retina, then in the normal eye the retina must be at the focal length of the cornea, which is 2 cm.

2. A
   The passage states in the first paragraph that the cornea is a converging lens. A converging lens has a positive focal length. To obtain the image distance i, we rearrange the equation:
   $$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

   Since f is positive, so is 1/f. A distant object means that o is large, and so 1/o is small. The right hand side therefore remains positive, and so i is positive. This implies that the image is real. The magnification, defined as $m = -i/o$, is negative in this case since i and o are positive and there is a negative sign in front. A negative magnification means that the image is inverted.

3. D
   Despite the note in the question stem that seems to direct you to use the equation given in the passage, the answer can actually be reached by qualitative reasoning alone. Myopia, as described in the passage, occurs when the cornea focuses the image of a distant object in front of the retina. The image distance is too short: the light converges too rapidly. In the case of hyperopia, the cornea focuses the image of a nearby object behind the retina: the image distance is too long. From the discussion to #1, we know that for distant objects, the image distance is equal to the focal length. We can thus conclude that myopia is corrected by a lens that would lead to an increased net focal length, while hyperopia is corrected by a lens that would lead to a reduced net focal length. From this alone, we can conclude that the lenses must be different in the two cases and eliminate choices A and C.

   For myopia, as pointed out above, the light is converging too rapidly. Placing another converging lens in the path of the light will only worsen the problem. We therefore need a diverging lens, which increases the net focal length of the system since it causes light to bend away from the normal. In hyperopia, the light does not converge rapidly enough: by the time it converges it is behind the retina. Putting a converging lens in front of the eye will therefore help solve the problem. So choice D is correct.

4. C
   The relevant equation is the one given in the passage:
   $$\frac{1}{f_c} + \frac{1}{f_l - x} = \frac{1}{i}$$

   where $f_c$ is the focal length of the cornea, $f_l$ the focal length of the correcting lens, x the distance between the two, and i the image distance measured from the cornea. We are given in the question stem that $f_c = 1.8$ cm, $f_l = -16.5$ cm, and $x = 1.5$ cm. To solve for i, we need to rearrange the equation:
   $$\frac{1}{i} = \frac{1}{f_c} + \frac{1}{f_l - x} = \frac{f_l - x + f_c}{f_c (f_l - x)}$$

   as developed by
Instead of performing a laborious division, notice that 18 is slightly greater than 16.2, and so the expression is equal to 1.8 times something slightly greater than 1. This should result in something slightly greater than 1.8, which makes choice C, 2.0 cm, the most sensible choice. (Choice D, 2.5 cm, is much too large. It is greater than 1.8 by 0.7, which is more than one-third of 1.8. We would have needed to multiply 1.8 by something greater than 1.33, which is not what we have here.)

5. D

This question again involves the equation given in the passage for a two-lens system:

\[
\frac{1}{f_c} + \frac{1}{f_l - x} = \frac{1}{i}
\]

In the present case, we have a simplification. Since we are dealing with a lens in contact with the cornea, the distance x between the lenses is zero. The equation then reduces to:

\[
\frac{1}{f_c} + \frac{1}{f_l} = \frac{1}{i}
\]

Therefore, if the focal length of the cornea doubles and the focal length of the contact lens doubles, the image distance would also double:

\[
\frac{1}{2f_c} + \frac{1}{2f_l} = \frac{1}{2} \left( \frac{1}{f_c} + \frac{1}{f_l} \right) = \frac{1}{2} \left( \frac{1}{i} \right) = \frac{1}{2i}
\]

6. A

When light travels through a medium its speed is characterized by the index of refraction of the medium: \( v = \frac{c}{n} \), where \( v \) is the speed of light in the medium, \( c \) is the speed of light in vacuum (\( \approx 3.0 \times 10^8 \) m/s), and \( n \) is the index of refraction (always greater than or equal to 1). The higher the index of refraction, the slower light travels in the medium. A change in the index of refraction also causes a change in the angle the light makes with the normal. The exact relationship is described by Snell’s law:

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

where \( \theta \) is the angle the light makes with the normal to the boundary. Qualitatively, a higher index of refraction causes light to be bent towards the normal, while a lower index of refraction causes it to be bent away from the normal. The fact that light bends towards the normal as it travels from air into glass indicates that the index of refraction of glass is greater than that of air.

Choice B is incorrect because the exact opposite is true. If light were to travel faster in glass than in air, it would mean that glass has a lower index of refraction, and light would then be bent away from the normal as it goes from air into glass.

Choice C is a false statement: the speed of light, as described above, is dependent on the medium in which it travels.

Choice D states that some of the light is reflected at the surface of the glass lens. This is a true statement, but is irrelevant to the bending of light, the phenomenon of refraction.

Passage II (Questions 7—12)

7. D

This is a perfect example of how useful dimensional analysis is as a tool. To obtain the answer, all we have to do is to determine which of the choices have matching units on both sides of the equation. More specifically, since \( c \), the speed of light, can be measured in units of m/s, we need only evaluate which of the combinations of the constants on the left hand side has units of length/time. The permittivity constant, \( \varepsilon_0 \), has units of \( \text{C}^2/\text{N}\cdot\text{m}^2 \), while the permeability constant has units of \( \text{N}^2\cdot\text{s}^2/\text{C}^2 \). We may notice that the coulomb squared is in the numerator of one and the denominator of the other; likewise for N.
If we want to cancel both (which we do if we want to end up with just m/s), we should therefore multiply the two: $\varepsilon_0 \times \mu_0$ has units of:

$$\frac{C^2}{N\cdot m^2} \times \frac{N\cdot s^2}{C^2} = \frac{s^2}{m^2}$$

This expression contains the same fundamental units as speed (length and time), but the combination is different. First, we want the length dimension to be on top and the time dimension to be on the bottom. We therefore need the reciprocal of the product of the two constants:

Units of $(\varepsilon_0 \times \mu_0) = \frac{s^2}{m^2}$

$\therefore$ Units of $\frac{1}{\varepsilon_0 \times \mu_0} = \frac{1}{s^2/m^2} = \frac{m^2}{s^2}$

This is however still not complete: instead of $m^2/s^2$, we would like m/s. This is easily taken care of by taking the square root:

Units of $\frac{1}{\varepsilon_0 \mu_0} = \frac{m^2}{s^2}$

Units of $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$

which is the unit of speed. One can easily verify that all of the other answer choices do not yield the right units. Note that in this case we need not pay attention to the numerical values of the constants. If one of the other answer choices had been, for example, $\frac{2}{\sqrt{\varepsilon_0 \mu_0}}$ or $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, which have the same units (since they differ only by dimensionless factors), then we would need to make use of the values to differentiate between the correct and incorrect choices.

8. C

The relationship between frequency and wavelength for electromagnetic radiation propagating through a vacuum is the same regardless of how the light is polarized: $f\lambda = v$, where $v = c = 3 \times 10^8$ m/s in vacuum. In this case, then, the frequency is:

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{600 \text{ nm}} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{-9} \text{ m}} = 5 \times 10^{15} \text{ s}^{-1} = 5 \times 10^{14} \text{ s}^{-1} = 5 \times 10^{14} \text{ Hz}$$

9. B

The relevant formula is given at the end of the second paragraph, giving the intensity of an electromagnetic wave. This, however, is only one part in arriving at the correct answer. Intensity, we are told in the passage, is power per unit area. So in order to determine the power transmitted, we need to multiply the intensity by the area “intercepting” the wave. In this question, the area is that of the disk, and so the result we are after can be obtained by:

$$P = r^2 \times I = r^2 \times \frac{E_{\text{rms}}^2}{c\mu_0} = 4 \times \frac{30^2}{(3 \times 10^8 \times 4 \times 10^{-7})}$$

Canceling the 4 from the numerator and the denominator gives us $P = 30^2/30 = 30$ J/s.

10. B

Statement I is true: The speed of light in a medium with index of refraction $n$ is given by $c/n$, where $c$ is the speed of light in vacuum. The frequency of the light, however, does not change. Since $v = f\lambda$, and $v$ changes but $f$ does not, the wavelength $\lambda$ then must also change. In particular, on going from a vacuum into a medium with index of refraction equal to $n$, the wavelength becomes $\lambda/n$. Hence statement II is also correct in the way it describes the dependency of the wavelength of a particular wave on the medium, but statement III is incorrect.
11. A

This question depends a great deal on reading comprehension. In the first paragraph of the passage, it is stated that the plane of polarization of a plane-polarized electromagnetic wave is defined by the electric field vector and the Poynting vector. The actual axis of polarization is taken to be the direction along which the electric field vector oscillates. Therefore, if the electric field vector oscillates in the vertical direction and the wave passes through a polarizing filter with horizontal polarizing direction, the electric field vector will have no component along the direction of the filter, and the resulting transmitted intensity will be zero.

12. C

Pressure is force per unit area, and we can think of the pressure as resulting from a force that light imparts onto a surface having a specific area. When the light hits the surface, some kind of collision takes place, and the momentum of the light changes. The force must be proportional to this change in momentum, from the “impulse formulation” of Newton’s second law:

\[ F = \frac{p}{t} \]

It says in the passage that when the light is totally absorbed by the surface, the radiation pressure equals \( I/c \), where \( I \) is the intensity and \( c \) is the speed of light. If the change in momentum is doubled when light is reflected, then, the force will be twice that for the case of absorption. Pressure, being force per unit area, will be doubled as well, becoming \( 2I/c \).

The fact that the change in momentum is doubled when light is reflected should make sense if one thinks of a billiard ball. If it collides with a wall and sticks, it has come to rest. The change in momentum, then, is equal to \( mv_{\text{final}} - mv_{\text{initial}} = 0 \) – \( mv_{\text{initial}} = -mv_{\text{initial}} \). However, if it rebounds with a speed equal to its initial speed (in the opposite direction), corresponding to total reflection, the change in momentum is then:

\[ mv_{\text{final}} - mv_{\text{initial}} = m(-v_{\text{initial}}) - mv_{\text{initial}} = -2mv_{\text{initial}} \]

which is twice the previous value.

13. B

Only a converging lens or mirror can produce a real image. This is because the light rays must converge at one point in order to focus the image. A convex mirror is a diverging mirror. It can never produce a real image regardless of whether the image is inverted or upright. So I cannot be a correct choice, which eliminates choices A and C. III, a concave lens, is a diverging lens, and therefore also cannot produce real images. This leaves II, and so B is the correct choice.

We may verify that concave mirrors can produce real inverted images. First, concave mirrors are convergent mirrors. They are therefore capable of producing real images. The object distance \( o \), the image distance \( i \), and the focal length \( f \) must satisfy the relation \( \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \). The equation can be rearranged to give:

\[ i = \frac{of}{(o - f)} \]

For a converging optic, \( f \) is positive, and so \( i \) is positive only if \( o > f \). I.e., the image is real if \( o > f \). If \( o < f \), the image distance is negative, and the image is virtual. Second, the magnification is given by \( m = -\frac{i}{o} = -\frac{f}{(o - f)} \). If \( o > f \), the magnification is negative and (for positive \( f \)) the image is inverted. Otherwise the magnification is positive and the image is upright. In summary, if the object is outside the focal length of the concave mirror, the image will be real and inverted. If the object is inside the focal length of the mirror, the image is virtual and upright. The following ray-tracing diagrams may be helpful:
14. **A**

A convex mirror always produces a virtual image because it is a diverging mirror. For mirrors, images that are virtual are always behind the mirror, where light never physically reaches. Therefore we can narrow our choices down to A and B. The optics equation gives us:

\[
\frac{1}{i} = \frac{1}{f} - \frac{1}{o}
\]

For a diverging optic (convex mirror or concave lens), \( f \) is negative. In this case, then, we have \( \frac{1}{i} \) equal to a negative number minus a positive number, which gives a negative number. The image distance therefore is also negative. The magnification, \( m \), which is \( -\frac{i}{o} \), will thus be a positive number. The image is upright, and so choice A is correct.

15. **C**

The angle of reflection is equal to the angle of incidence, where both are measured from the normal to the boundary. Therefore, if we know the angle of incidence, we would have the angle of reflection. Unfortunately, the angle of incidence is not one of the choices. What we need, then, is information that would enable us to determine what the angle of incidence is. Snell’s law states that when light travels from medium 1 into medium 2, the angles it makes with the normal to the boundary satisfy:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

where \( n_1 \) is the index of refraction of medium 1, \( \theta_1 \) is the angle of incidence, \( n_2 \) is the index of refraction of medium 2, and \( \theta_2 \) the angle of refraction. We can rearrange the equation to give:

\[ \sin \theta_1 = \frac{n_2}{n_1} \sin \theta_2 \]

To know the sine of the angle of incidence (which would then give us the angle itself), then, we would need to know the ratio of the indices of refraction and the (sine of) the angle of refraction. The former is known according to the question stem. The latter is choice C. Once we calculate the sine of the angle of incidence, we know the angle of incidence itself.

16. **D**

This question asks about polarizing lenses. When light is plane-polarized, the electric field vectors of all the light waves lie in the same plane. A polarizing lens only allows the component of the electric field vector that is parallel or antiparallel to the polarization to pass through it. So the intensity of the light that passes through a polarizing lens is diminished when the electric field vectors are not parallel or antiparallel to the polarization axis. If they are perpendicular to it, then no light will make it through at all.
In this question, we have two polarizing lenses that are aligned so that the intensity of the transmitted light is a maximum. This will occur when the polarization axis of the second lens is parallel (or antiparallel which is the same thing) to the polarization axis of the first lens. Originally, let’s say that both lenses polarize light along the y-axis. So light that has its electric field oscillating in the vertical direction will pass through both with maximum intensity.

First lens  Electric field  Second lens  Electric field
after first lens  \( E_1 \)  after second lens  \( E_2 = E_1 \)

Now the first lens is rotated \( 45^\circ \) in the clockwise direction. This axis indicates the direction of the electric field vectors that are allowed to pass through the first lens with maximum intensity. The second lens, however, will now filter out some of the light that made it through the first lens, letting through only that component that is oriented along the y axis, which is \( \frac{1}{2}\sqrt{2} \) the magnitude of the light going in.

First lens  Electric field  Second lens  Electric field
after first lens  \( E_1 \)  after second lens  \( E_2 = E_{1y} = E_1 \sin 45^\circ \)

Our goal is to rotate the second lens in a counterclockwise manner so that the intensity of the light transmitted through the two-lens system is again at a maximum. This occurs when the polarizing axes of the two lenses are again aligned. We can accomplish this by also rotating the second lens by \( 45^\circ \) clockwise, but we are asked specifically for counterclockwise rotation: a counterclockwise rotation of \( 45^\circ \) would make the two axes perpendicular, letting no light pass through. With the counterclockwise constraint, then, the only choices are \( (180^\circ - 45^\circ) = 135^\circ \), or \( (360^\circ - 45^\circ) = 315^\circ \). Rotating counterclockwise by \( 135^\circ \) would make the two antiparallel, but this does not matter since the field vectors oscillate up and down anyway (i.e. there really is no parallel vs. antiparallel distinction). Choice D is therefore correct.

17. B

This question asks why images of stars produced by telescopes are surrounded by blurry rainbow-colored fringes. Let us approach the question by evaluating each explanation given in the answer choices.

Choice A states that different colors of light travel at different speeds in vacuum. This is incorrect because different colors of light differ in their frequency, and in a vacuum, all light, regardless of frequency, travel at the speed of \( c = 3.0 \times 10^8 \) m/s. Without worrying about whether this makes a good explanation, we can reject it for being a false statement. (Note that in materials with index of refraction greater than one, however, light of different frequencies may actually “see” slightly different indices of refraction, which would lead to slightly different speeds.)

Choice B states that lenses bend different colors of light through different angles. When sunlight passes through a glass prism, it is separated into its component colors. This is because when light passes from air into glass different frequencies or colors of light are bent at different angles. (This again has to do with the fact that different frequencies of light may see slightly varying indices of refraction in the same medium. Snell’s law would then predict that they would be bent at slightly different angles.) This fact would seem to account for the appearance of blurry rainbow colored fringes around images of stars produced by refracting telescopes.

Choice C states that the colored fringes appear because the stars are far away. We have no evidence on which to base this conclusion. We are not told that there are more such fringes around images of stars that are farther away, nor are we told anything else that would indicate a relationship between the distance the stars are from the Earth and the appearance of colored fringes.

Choice D states that the Earth’s atmosphere changes the apparent color of a star. Even if this were true, it would not explain the appearance of fringes: it would just account for one color—that of the image. It turns out that dust and impurities in the Earth’s atmosphere can cause scattering which allows more of the red component of starlight to penetrate the atmosphere, but this still does not explain the existence of multicolored fringes seen around the images of stars.

18. D
Total internal reflection means that all the light incident on the boundary between medium 1 and medium 2 is reflected back into medium 1. No light travels into medium 2; there is no refracted ray. At this point we can rule out choices B and C because they both refer to an angle of refraction. Since there is no refracted ray, there can be no angle of refraction.

Why would there be no refracted ray? Snell’s law describes the general relationship between the angle of incidence and the angle of refraction:

$$\sin \theta_1 = \frac{n_2}{n_1} \sin \theta_2$$

If $n_2 < n_1$, $\sin \theta_2$ would be larger than $\sin \theta_1$, and since all angles are < 90°, $\theta_2$ would be larger than $\theta_1$: the refracted ray is bent farther away from the normal in medium 2.

As the angle of incidence keeps increasing, the refracted ray will be bent more and more away from the normal, until $\theta_2$ reaches 90°, in which case the refracted ray is just skimming the boundary:

If the angle of incidence were to increase any more, the “refracted” ray will actually be going back into medium 1, i.e. the incident ray is reflected internally. This threshold value of $\theta_1$ for total internal refraction to occur can be solved using Snell’s law, setting $\theta_2$ equal to 90°:

$$\sin \theta_1 = \frac{n_2}{n_1} \sin 90° = \frac{n_2}{n_1}$$

$$\theta_1 = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

Angles of incidence greater than this value would then lead to total internal reflection. Note that this phenomenon occurs only when light goes from a medium with a higher index of refraction into one with a lower index of refraction.